友万科技

高级计量经济学及Stata应用

分位数回归：横截面、面板与IV估计

陈强

山东大学经济学院

qiang2chen2@126.com

公众号/网站：econometrics-stata

网易云课堂：http://study.163.com/u/metrics

本讲内容

1. 分位数回归（横截面）

2. 分位数处理效应（工具变量法）

3. 面板分位数回归

1. 分位数回归

- 在迄今为止的回归模型中，我们着重考察解释变量 $\boldsymbol{x}$ 对被解释变量 y 的条件期望 $\mathrm{E}(y\,|\,\mathbf{x})$ 的影响，实际上是均值回归。

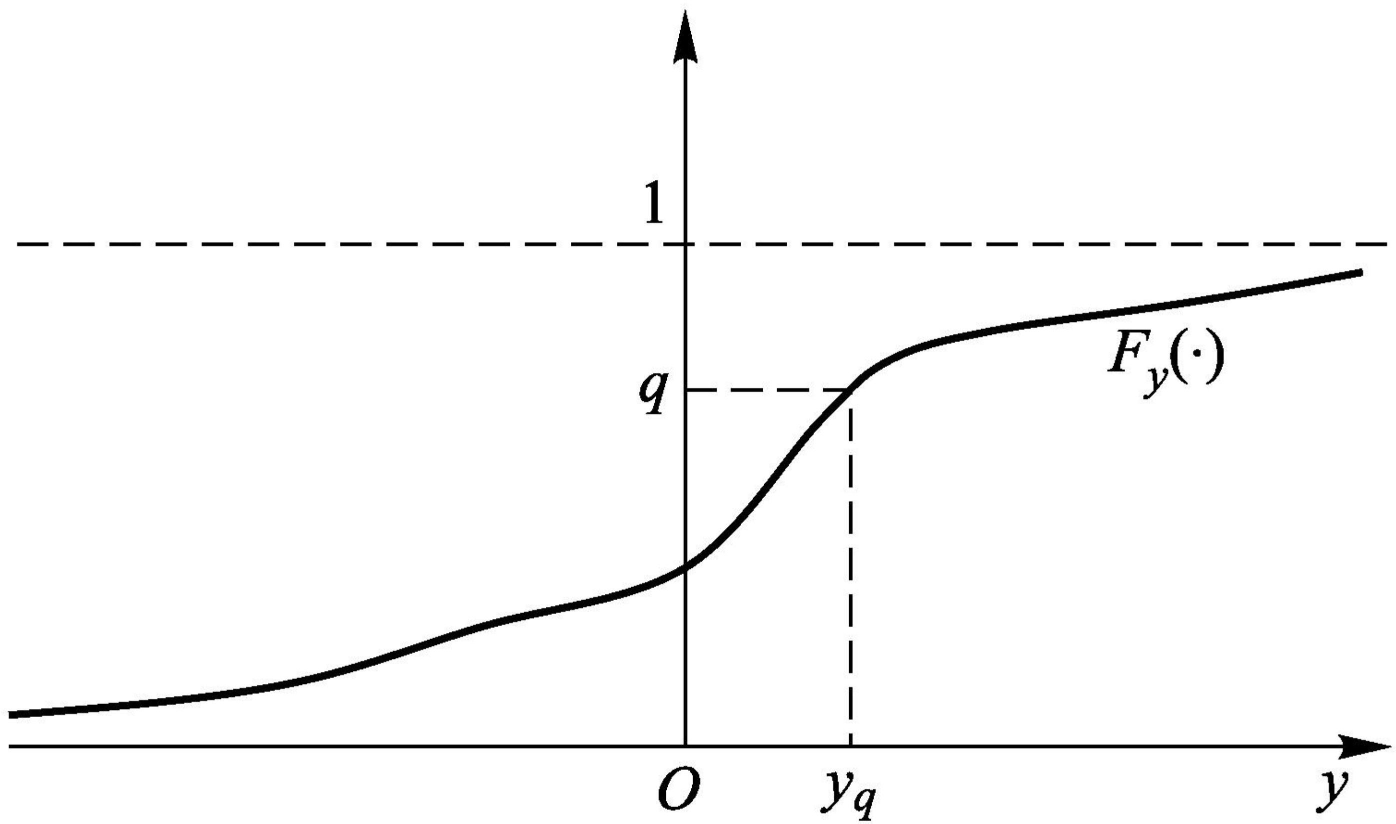
- 但真正关心的是 $\boldsymbol{x}$ 对整个条件分布 $y\,|\,\mathbf{x}$ 的影响，而条件期望只是刻画条件分布的一个指标而已。

- 使用OLS的“均值回归”，由于最小化的目标函数为残差平方和，故易受极端值(outliers)的影响

为何需要分位数回归

- Koenker and Bassett(1978)提出“分位数回归”(Quantile Regression，简记QR)
- 使用残差绝对值的加权平均 (比如, $\sum_{i=1}^{n}|e_i|$)作为最小化的目标函数，不易受极端值影响，更稳健
- 分位数回归还能提供关于条件分布 $y|\mathbf{x}$ 的全面信息。比如，估计出条件分布的若干重要的条件分位数(conditional quantiles)，比如中位数(median)、1/4分位数(lower quartile)、3/4分位数(upper quartile)

总体分位数



总体q分位数与累积分布函数

总体分位数的定义

- 假设Y为连续型随机变量，其累积分布函数为 $F_y(\cdot)$，则Y的“总体*q*分位数”(population *q*th quantile，0<*q*<1)，记为y_q，满足以下定义式：

$$q = \mathrm{P}(Y \le y_q) = F_y(y_q)$$

- 总体*q*分位数 y_q 正好将总体分布分为两部分，小于或等于 y_q 的概率为*q*，而大于 y_q 的概率为 1-*q*

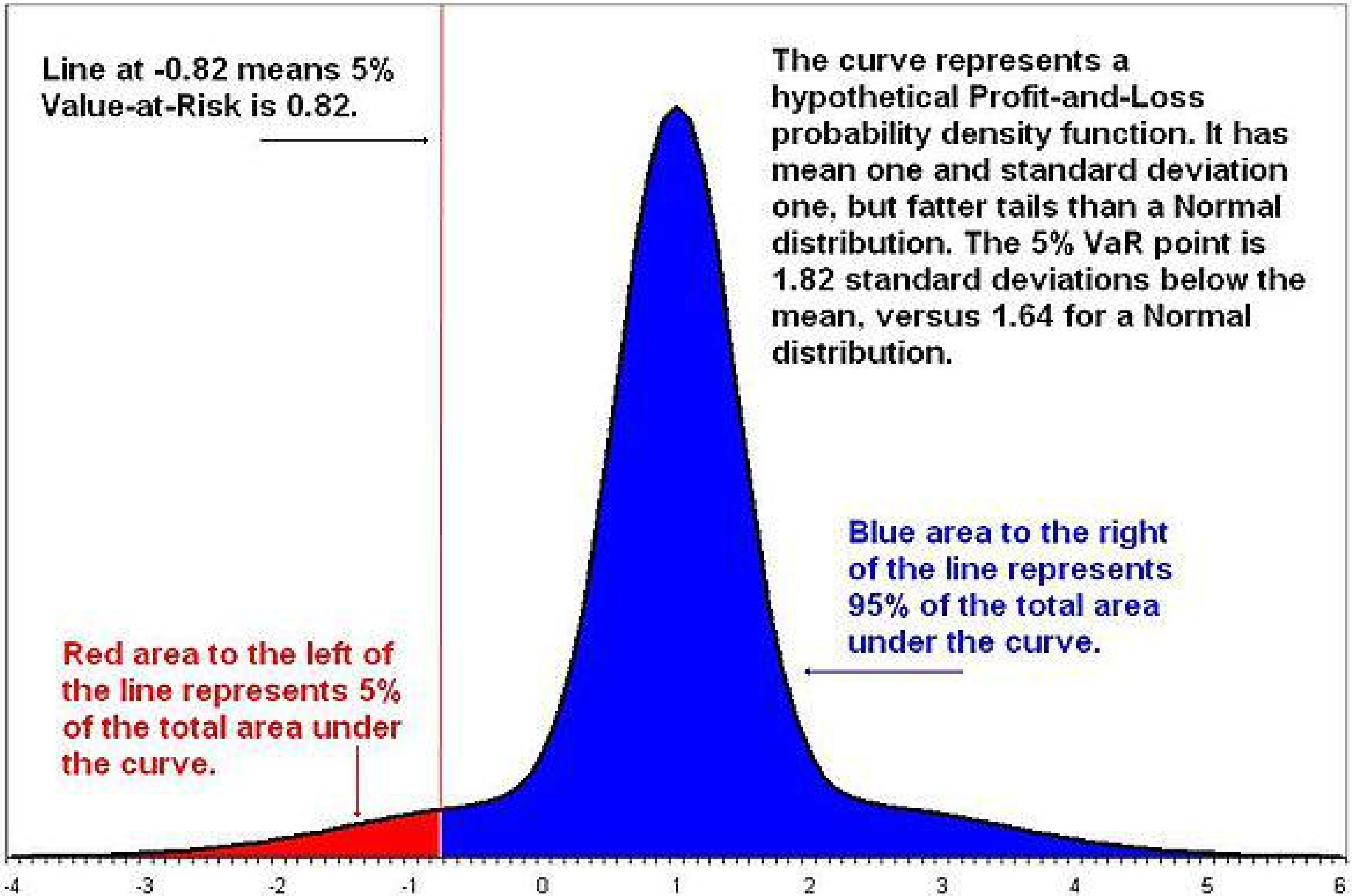
- 如果 $F_y(\cdot)$ 严格单调递增，则有 $y_q = F_y^{-1}(q)$

总体分位数的应用

- 显著性水平为0.05的*F*检验之**临界值**
  = 此*F*分布的0.95分位数

- 显著性水平为0.05的双边*t*检验之**临界值**
  = 此*t*分布的0.975分位数

- 金融资产的Value at Risk (VaR)


Line at -0.82 means 5% Value-at-Risk is 0.82.
The curve represents a hypothetical Profit-and-Loss probability density function. It has mean one and standard deviation one, but fatter tails than a Normal distribution. The 5% VaR point is 1.82 standard deviations below the mean, versus 1.64 for a Normal distribution.
Blue area to the right of the line represents 95% of the total area under the curve.
Red area to the left of the line represents 5% of the total area under the curve.
-4
-3
-2
-1
0
1
2
3
4
5
6

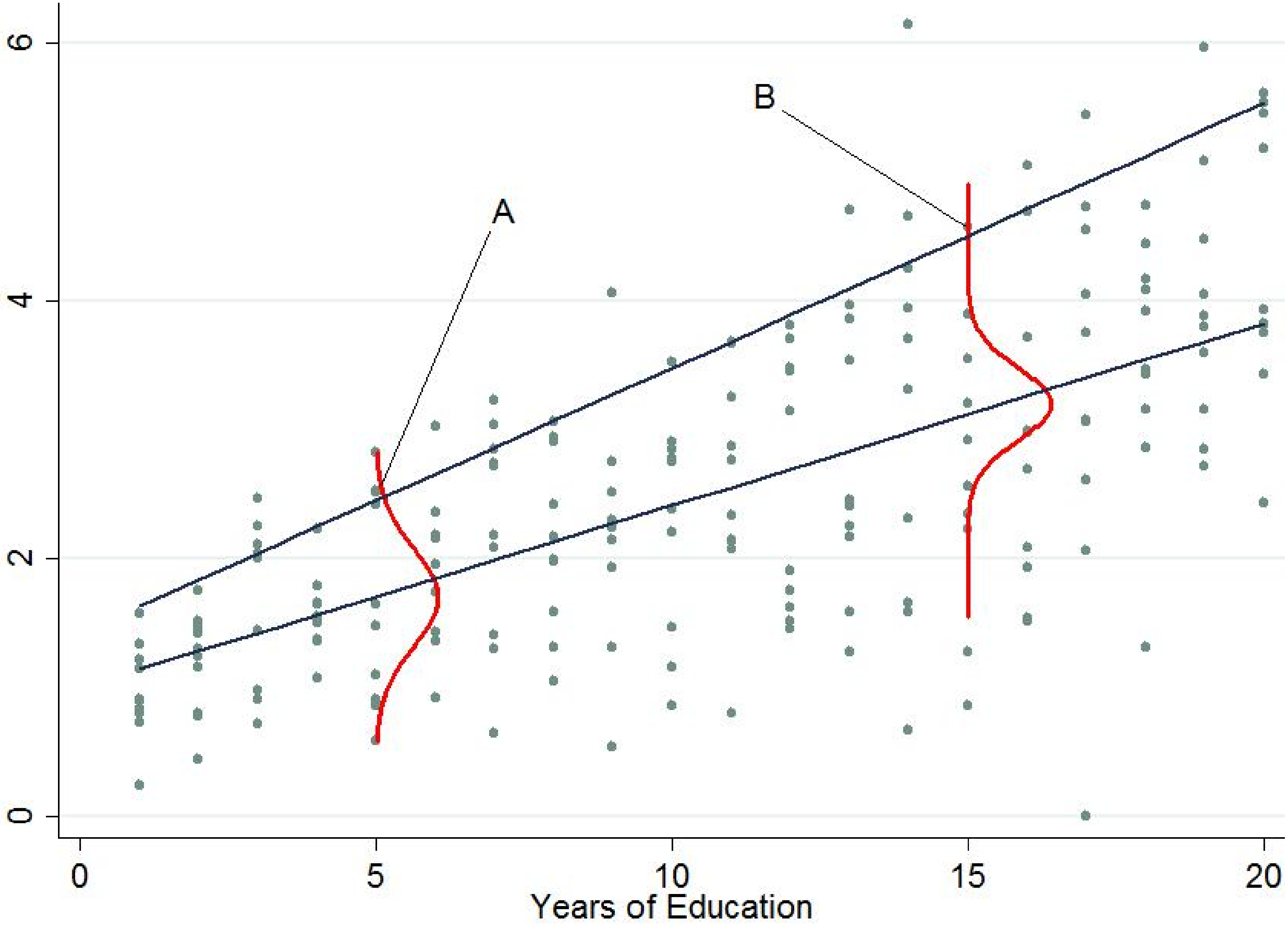
条件分布的总体分位数

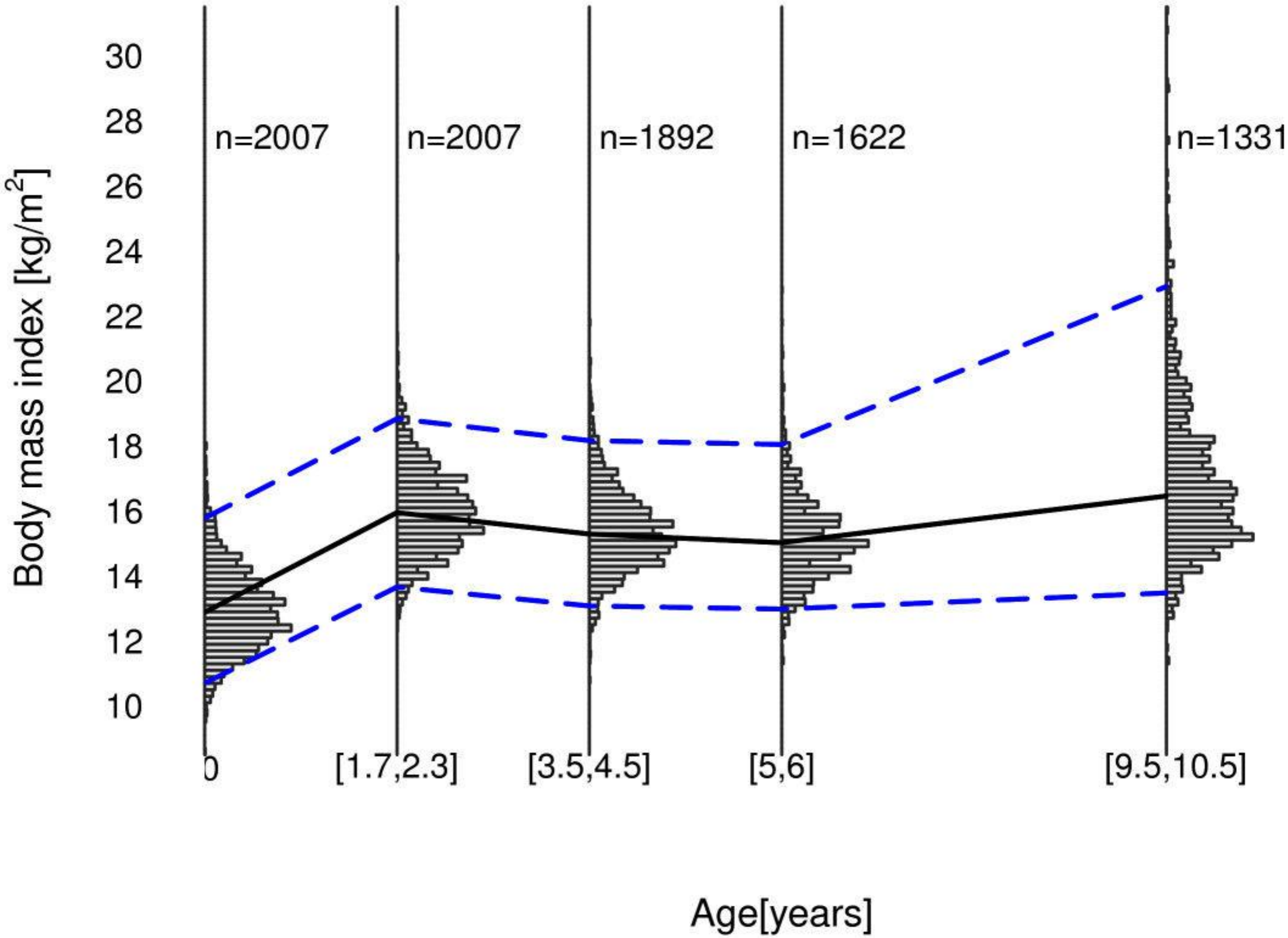
- 条件分布 $y|\mathbf{x}$ 的总体 q 分位数，记为y_q ，满足以下定义式：

$$q = F_{y|\boldsymbol{x}}(y_q)$$

- 由于条件累积分布函数$F_{y|\boldsymbol{x}}(\cdot)$ 依赖于$\boldsymbol{x}$，故条件分布 $y|\mathbf{x}$ 的总体 q 分位数也依赖于$\boldsymbol{x}$，可写为 $y_q(\boldsymbol{x})$ ，称为“条件分位数函数”(conditional quantile function)。

- 对于线性回归模型，如扰动项为同方差，或异方差为乘积形式，则 $y_q(\boldsymbol{x})$ 是 $\boldsymbol{x}$ 的线性函数(见下页)


A
B
0
2
4
6
0
5
10
15
20
Years of Education



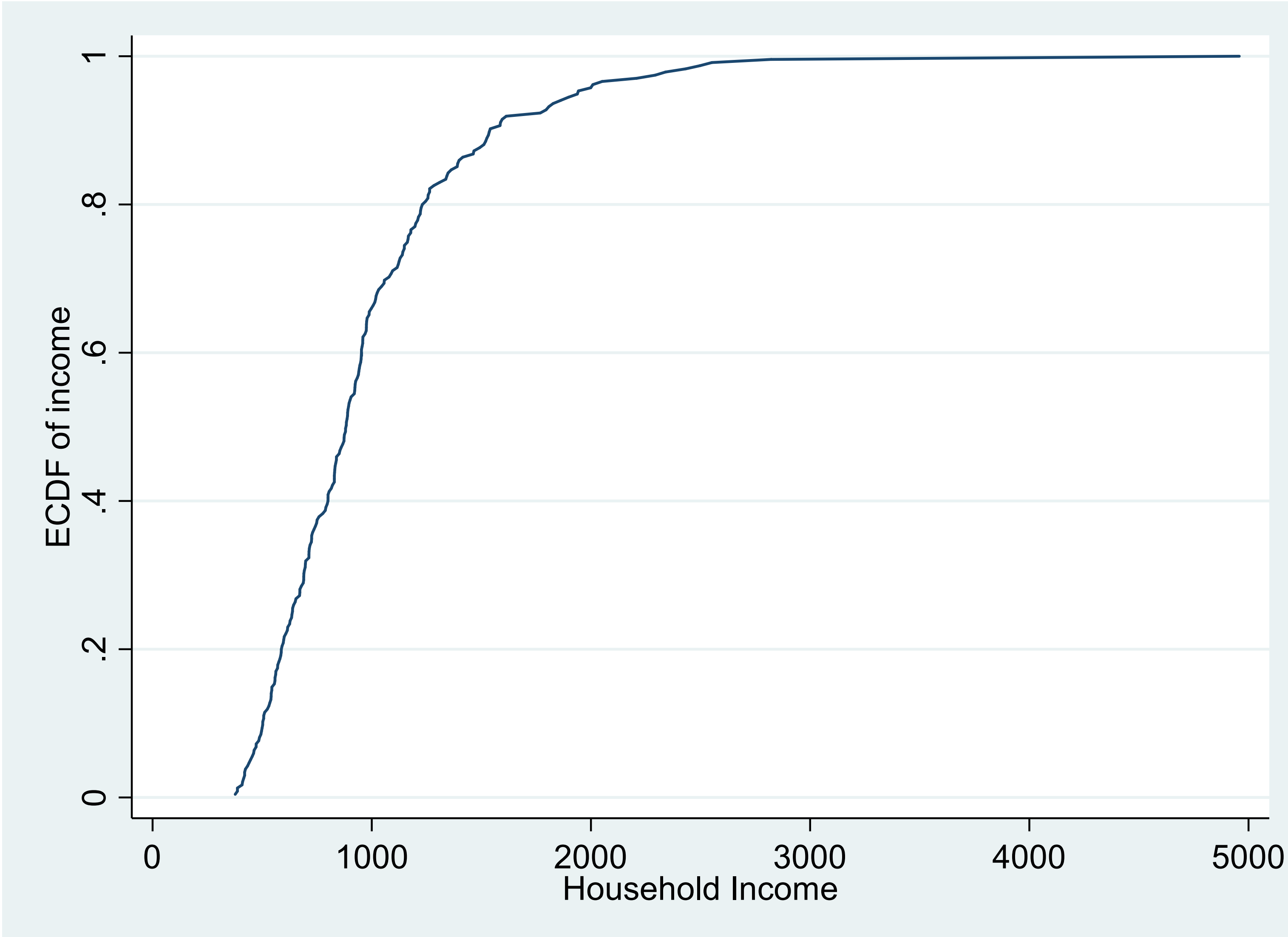
Body mass index [kg/m2]
30
28
26
24
22
20
18
16
14
12
10
n=2007
n=2007
n=1892
n=1622
n=1331
0
[1.7,2.3]
[3.5,4.5]
[5,6]
[9.5,10.5]
Age[years]

样本分位数

- 对于随机变量 *Y*，如果总体的*q*分位数 y_q 未知，则可以使用样本*q*分位数 $\hat{y}_q$ 来估计
- 首先将样本数据 $\{y_1, y_2, \cdots, y_n\}$ 按照从小到大的顺序排列为 $\{y_{(1)}, y_{(2)}, \cdots, y_{(n)}\}$，则 $\hat{y}_q$ 等于第[*nq*]个最小观测值，其中*n*为样本容量，[*nq*]表示大于或等于*nq*而离*nq*最近的正整数。
- 比如 *n*=97，*q*=0.25，则[*nq*]=[97x0.25]=25

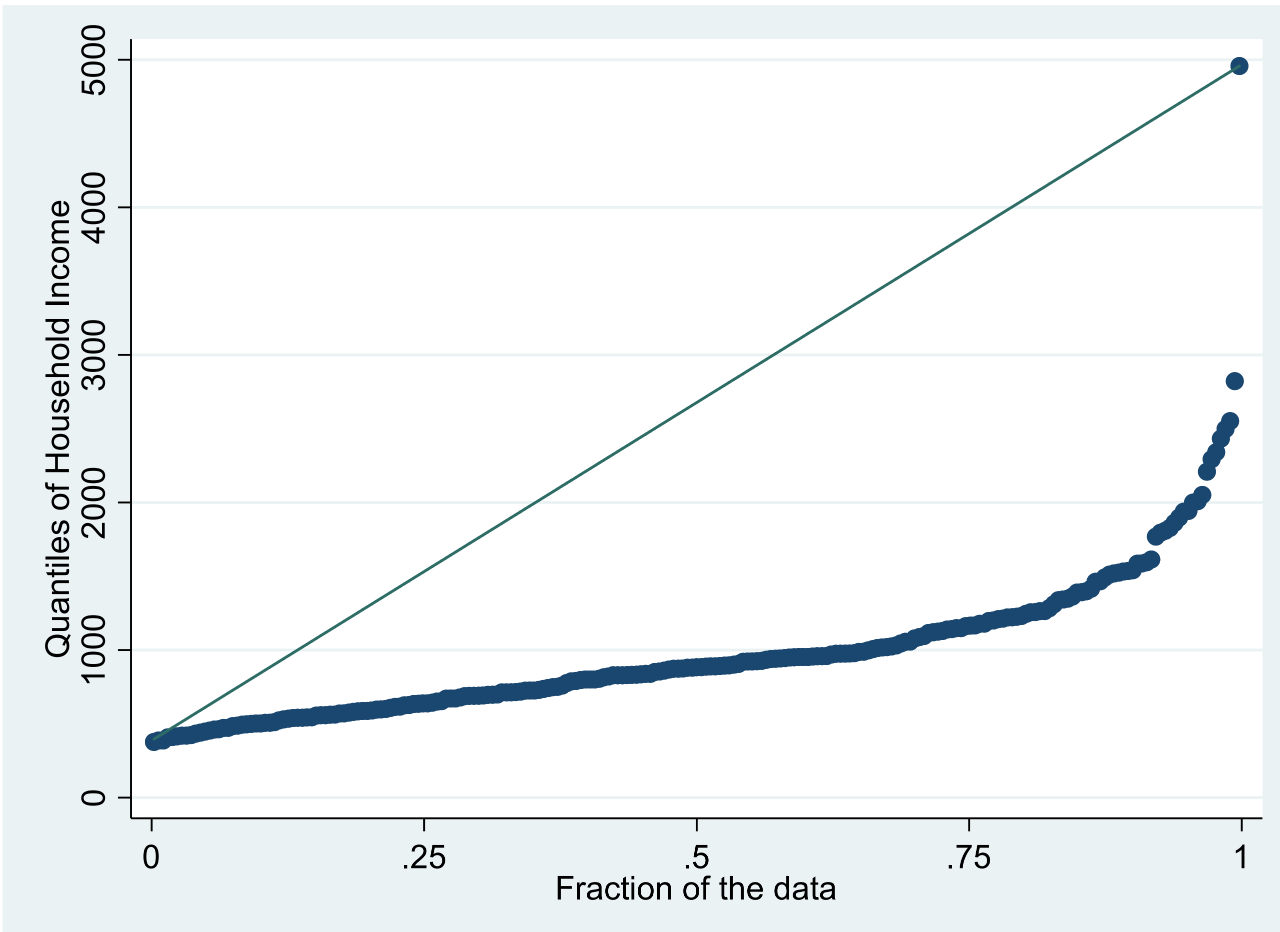
例：Engle (1857)

- 使用Engle(1857)的经典数据集，包括两个变量：income (家庭收入)，food (食物开支)
- 计算 income 的经验累积分布函数(empirical cdf)，并记为cdf_income，然后画图
- `use engle1857.dta`
- `cumul income, gen(cdf_income)`
- `line cdf_income income, sort`


ECDF of income
0
.2
.4
.6
.8
1
0
1000
2000
3000
4000
5000
Household Income

画分位数函数图

- quantile income



回归模型的样本分位数

- 通过排序计算样本分位数的方法不易推广到回归模型。
- 一种等价方法是，将样本分位数看成是某个最小化问题的解。
- 事实上，样本均值也可看成是最小化残差平方和问题的最优解

$$\min_{\mu} \quad \sum_{i=1}^{n}(y_i-\mu)^2 \quad \Rightarrow \quad \mu=\bar{y}\equiv\frac{1}{n}\sum_{i=1}^{n}y_i$$

回归模型的样本分位数（续）

- 样本中位数可视为是“最小化残差绝对值之和”问题的最优解，即

$$\min_{\mu} \quad \sum_{i=1}^{n} |y_i - \mu| \quad \Rightarrow \quad \mu = \text{median}\{y_1, y_2, \cdots, y_n\}$$

- 因为只要上式中 μ 的取值偏离中位数，就会使得残差绝对值之和上升

回归模型的样本分位数（续2）

友万科技

- 可将样本*q*分位数视为以下最小化残差绝对值的加权平均问题的最优解

$$\min_{\mu} \sum_{y_i \geq \mu} q\left|y_i - \mu\right| + \sum_{y_i < \mu} (1-q)\left|y_i - \mu\right| \quad \Rightarrow \quad \mu = \hat{y}_q$$

- 如果*q* =1/4，则满足“$y_i \geq \mu$”条件的观测值只得到1/4 的权重，而满足“$y_i < \mu$”条件的其余观测值则得到 3/4 的权重。因为估计的是1/4分位数(位于总体底部)，故较大观测值得到较小权重，而较小观测值得到较大权重。

分位数回归模型

- 假设条件分布 $y \mid \mathbf{x}$ 的总体q分位数 $y_q(\boldsymbol{x})$ 是 $\boldsymbol{x}$ 的线性函数：

$$y_q(\boldsymbol{x}_i) = \boldsymbol{x}_i'\boldsymbol{\beta}_q$$

- 其中，$\boldsymbol{\beta}_q$ 称为“q分位数回归系数”，其估计量可以由以下最小化问题来定义：

$$\min_{\boldsymbol{\beta}_q} \sum_{y_i \geq \boldsymbol{x}_i'\boldsymbol{\beta}_q} q\left|y_i - \boldsymbol{x}_i'\boldsymbol{\beta}_q\right| + \sum_{y_i < \boldsymbol{x}_i'\boldsymbol{\beta}_q} (1-q)\left|y_i - \boldsymbol{x}_i'\boldsymbol{\beta}_q\right|$$

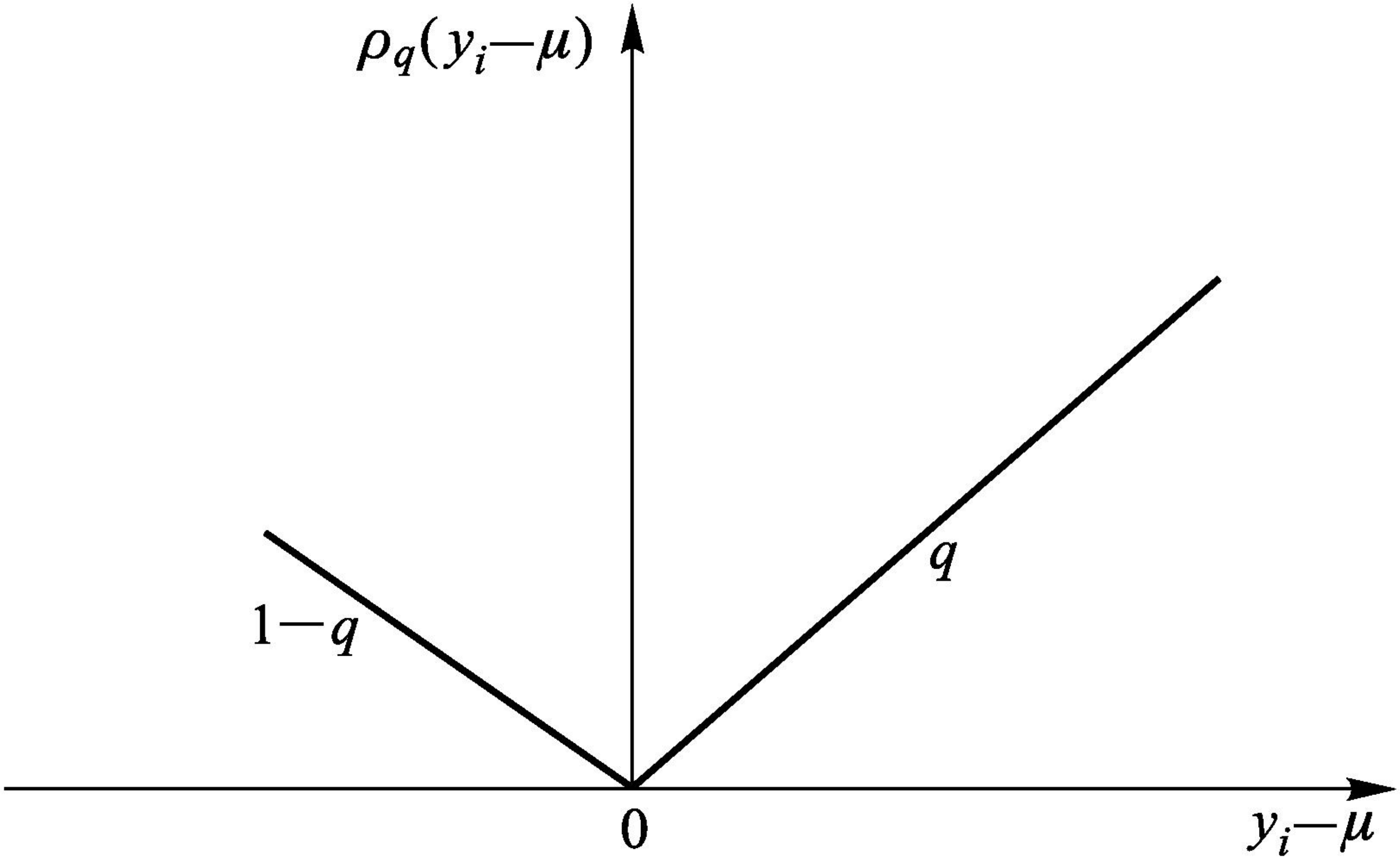
打钩函数（Check Function）

- 可将最小化问题写为

$$\min_{\boldsymbol{\beta}_q} \sum_{i=1}^{n} \rho_q(y_i - \boldsymbol{x}_i'\boldsymbol{\beta}_q)$$

- 其中，打钩函数为

$$\rho_q(u) = u(q - \boldsymbol{1}(u < 0)) = \begin{cases} qu & \text{if } u \geq 0 \\ (q-1)u & \text{if } u < 0 \end{cases}$$


$\rho_q(y_i - \mu)$
$1-q$
q
0
$y_i - \mu$

中位数回归

- 如果 q =1/2，则为“中位数回归”(median regression)。此时，目标函数简化为

$$\min_{\boldsymbol{\beta}_q} \sum_{i=1}^{n} \left| y_i - \boldsymbol{x}_i' \boldsymbol{\beta}_q \right|$$

- 故中位数回归也被称为“最小绝对离差估计量”(Least Absolute Deviation Estimator，简记LAD)。它比均值回归(OLS)更不易受到极端值的影响，故更加稳健

计算方法及性质

- 由于分位数回归的目标函数带绝对值，不可微，故常使用线性规划(linear programming)来计算

- 样本分位数回归系数 $\hat{\boldsymbol{\beta}}_q$ 是一致估计，且服从渐近正态分布：

$$\sqrt{n}(\hat{\boldsymbol{\beta}}_q - \boldsymbol{\beta}_q) \xrightarrow{d} N\left(\mathbf{0}, \mathrm{Avar}(\hat{\boldsymbol{\beta}}_q)\right)$$

- 可用公式计算标准误(默认)，或使用自助标准误(推荐)

QR的应用：Extremal Quantile

- **Conditional Value-at-Risk**: Forecast or explain *low quantiles* of future portfolio returns of an institution, *Y* , using current information, ***X***

- **Determinants of Birthweights**: How smoking, absence of prenatal care etc. during pregnancy affect *low birthweights*.

- **Stochastic Production Frontiers** Given production cost , we are interested in the highest production levels that only a small fraction of most efficient firms can attain.

分位数回归的Stata命令

(1) 只作一个分位数回归，使用默认的标准误

```
qreg y x1 x2 x3
qreg y x1 x2 x3,q(#)
```

(默认为中位数回归)
(#分位数回归)

(2) 只作一个分位数回归，使用自助标准误

```
set seed #
bsqreg y x1 x2 x3,reps(#) q(#)
```

(指定随机数的种子)

(自助法重复#次，#分位数回归)

分位数回归的Stata命令（续）

(3) 同时作多个分位数回归 (simultaneous quantile regressions)，使用自助法计算协方差矩阵

```
sqreg y x1 x2 x3,q(.25 .5 .75) reps(#)
```

(同时计算0.25，0.5与0.75分位数回归，自助法重复#次)

```
test [q25=q50=q75]: x1
```

(检验这三个分位数回归x1的系数是否相等)

分位数回归的Stata命令（续2）

友万科技

(4) 将不同分位数回归的系数及其置信区间进行画图比较

下载非官方命令

`net install grqreg.pkg`　(下载安装命令grqreg)

```
set seed #
bsqreg y x1 x2 x3,reps(#) q(.5)
```

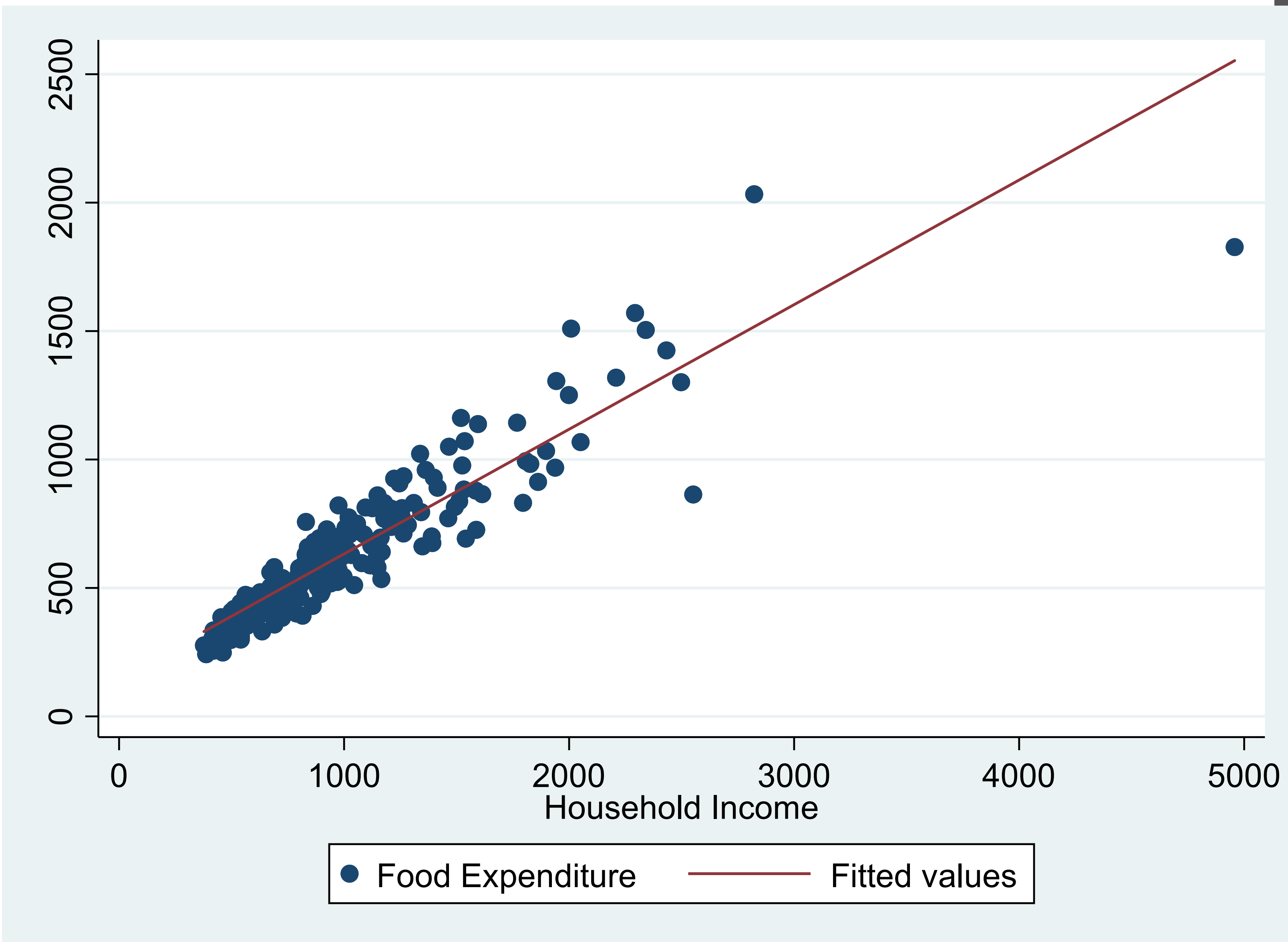
(为得到自助标准误而先作中位数回归)

```
grqreg,cons ci ols olsci
```

选择项“cons”表示也比较常数项，“ci”表示包括估计系数的95%置信区间，“ols”表示提供OLS估计系数作为参照系，“olsci”表示提供OLS系数的95%置信区间。

案例 1、Engle(1857)

- `use engle1857.dta,clear`
- 先看散点图与OLS回归
- `scatter food income || lfit food income`



0
500
1000
1500
2000
2500
0
1000
2000
3000
4000
5000
Household Income
Food Expenditure
Fitted values

同时进行0.1,0.5与0.9分位数的回归

- `set seed 123456`
- `sqreg food income,q(.1 .5 .9) reps(500)`

```
Simultaneous quantile regression                    Number of obs =        235
  bootstrap(500) SEs                                .10 Pseudo R2 =     0.4948
                                                    .50 Pseudo R2 =     0.6206
                                                    .90 Pseudo R2 =     0.7647
```

food	Coef.	Bootstrap Std. Err.	t	P>|t|	[95% Conf.	Interval]
q10						
income	.4017658	.0466956	8.60	0.000	.3097662	.4937653
_cons	110.1416	33.06357	3.33	0.001	44.99981	175.2833
q50						
income	.5601806	.0331924	16.88	0.000	.4947849	.6255762
_cons	81.48225	26.18517	3.11	0.002	29.89228	133.0722
q90						
income	.6862995	.025075	27.37	0.000	.6368968	.7357022
_cons	67.35087	20.44164	3.29	0.001	27.0768	107.6249

分位数回归系数的跨方程检验

- test [q10=q50=q90]: income

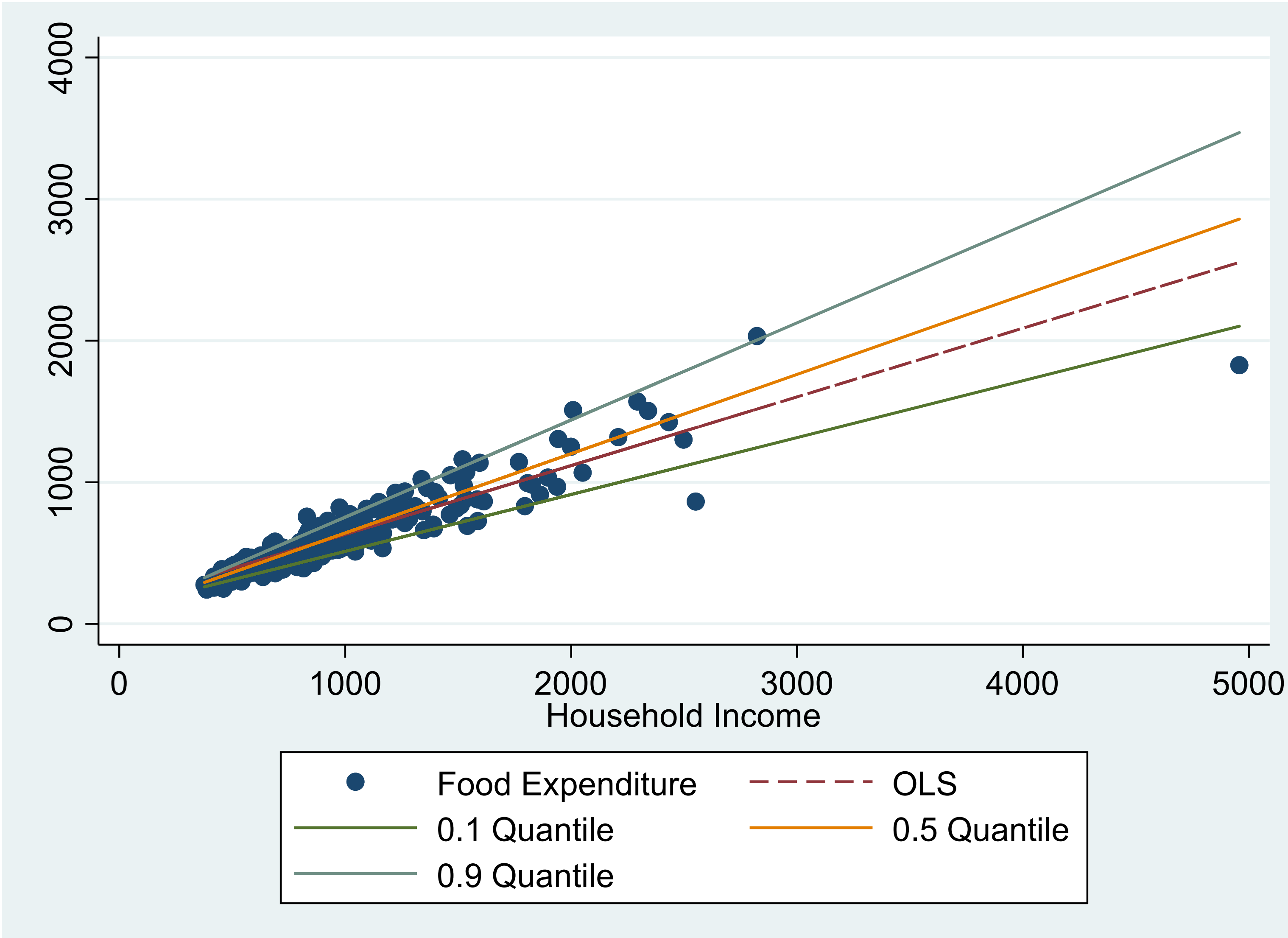
```
. test [q10=q50=q90]: income

 ( 1)  [q10]income - [q50]income = 0
 ( 2)  [q10]income - [q90]income = 0

       F(  2,   233) =   16.97
            Prob > F =    0.0000
```

Conditional Quantile Function的可视化

友万科技

- predict q10_h,equ(q10)
- label variable q10_h "0.1 Quantile“
- predict q50_h,equ(q50)
- label variable q50_h "0.5 Quantile“
- predict q90_h,equ(q90)
- label variable q90_h "0.9 Quantile“
- qui reg food income
- predict ols_h
- label variable ols_h "OLS“

- scatter food income||line ols_h income,lp(dash)||line q10_h income||line q50_h income || line q90_h income

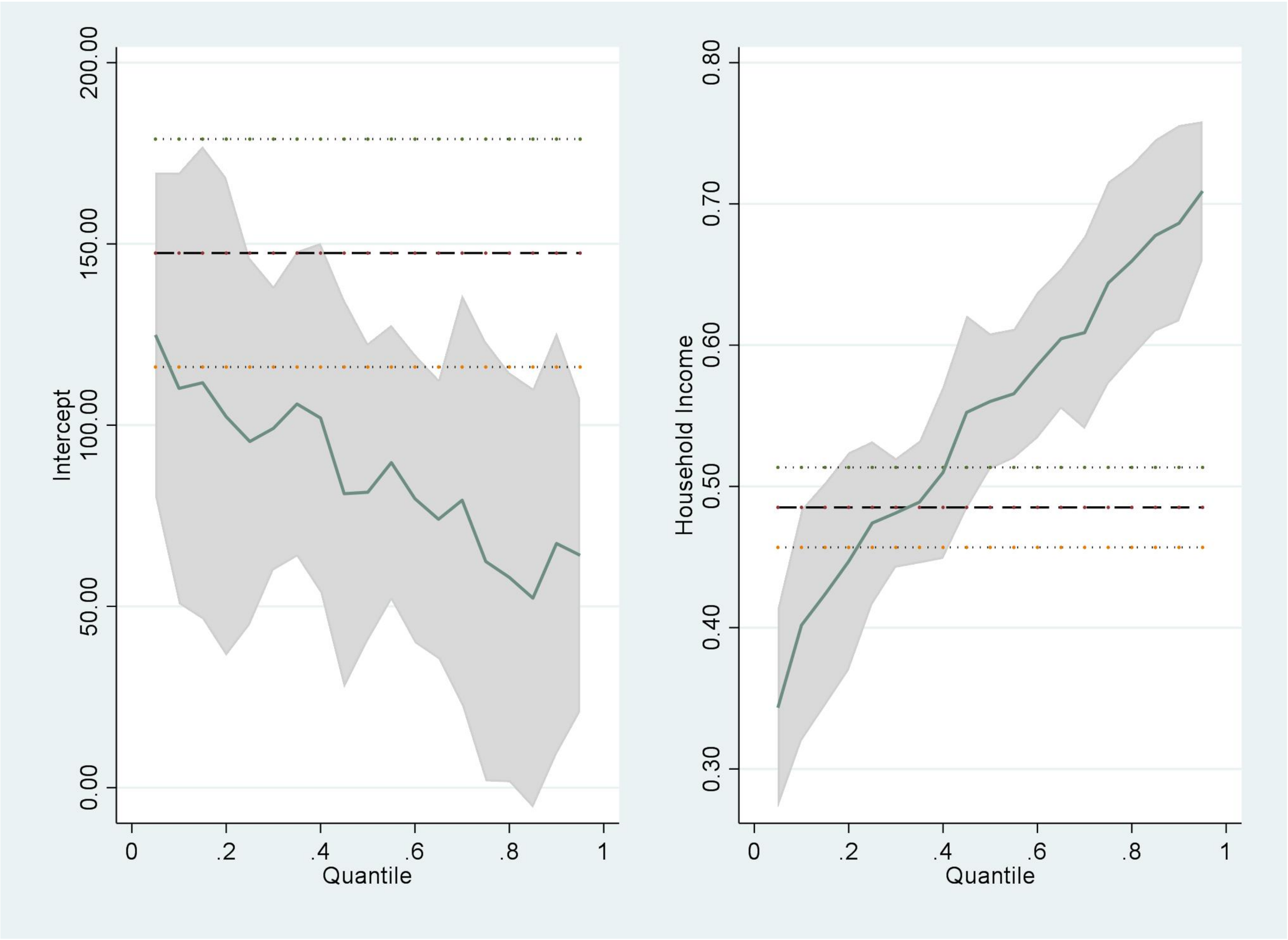


4000
3000
2000
1000
0
0
1000
2000
3000
4000
5000
Household Income
Food Expenditure
OLS
0.1 Quantile
0.5 Quantile
0.9 Quantile

分位数回归系数作为分位数的函数

- `qui bsqreg food income,q(.5) reps(500)`

- `grqreg,cons ci ols olsci seed(123456)`


200.00
150.00
100.00
50.00
0.00
Intercept
0
.2
.4
.6
.8
1
Quantile
0.80
0.70
0.60
0.50
0.40
0.30
Household Income
0
.2
.4
.6
.8
1
Quantile

系数解读

- 随着分位数的增加，收入(income)的分位数回归系数单调上升
- 增加收入对于食物支出(food)之条件分布的高分位数影响更大
- 异方差：食物支出的条件方差随着收入而增大
- **备注**：分位数回归系数仅度量 $\boldsymbol{x}$ 对条件分布 $y\,|\,\mathbf{x}$ 的作用，不能解读为对个体的影响

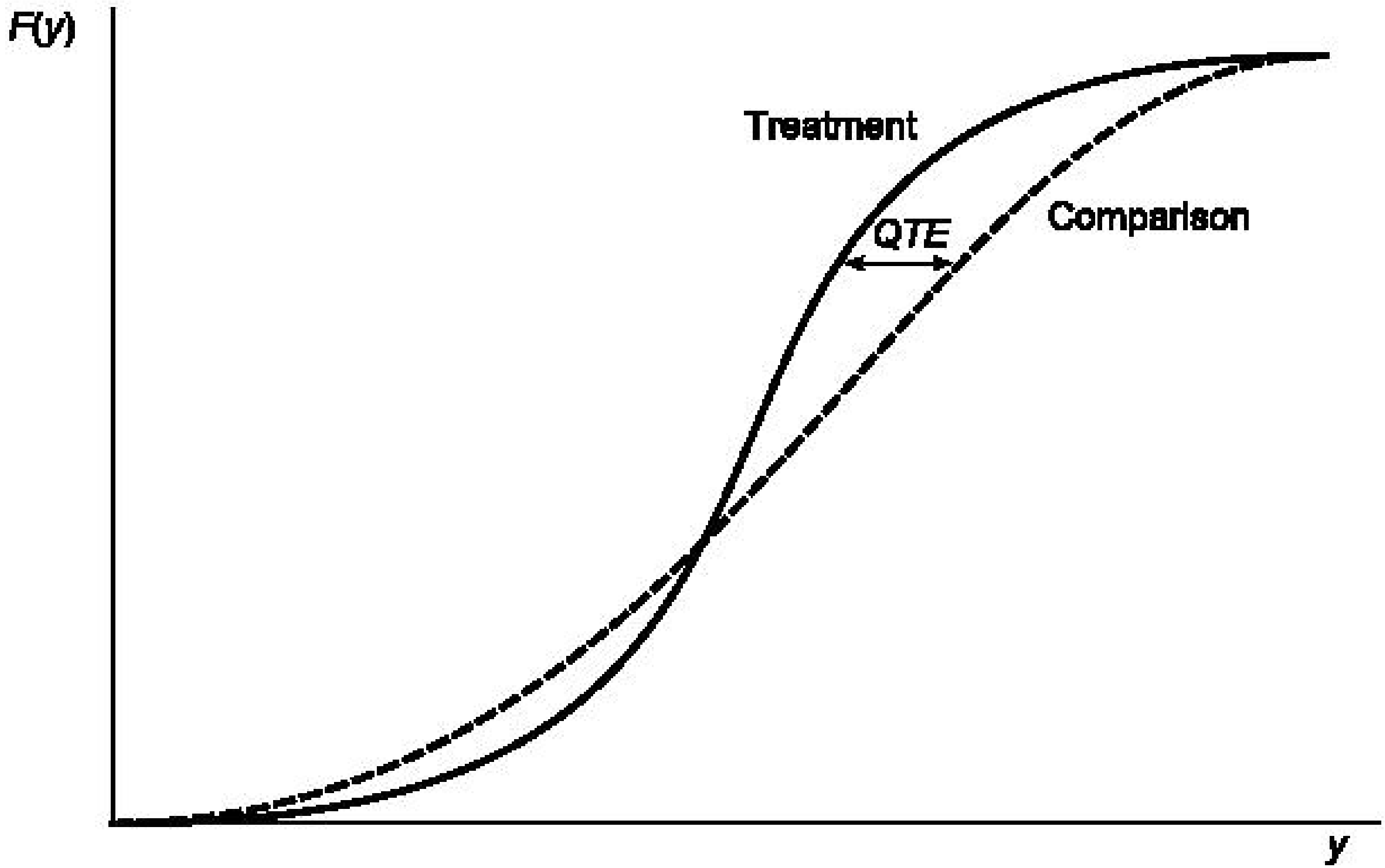
2. 分位数处理效应(QTE)

- 处理变量*D*对于结果变量*y*在不同分位数上的作用

$$\Delta_\tau = Q_{y(1)}(\tau) - Q_{y(0)}(\tau)$$

- *y*(1): Outcome with treatment (*D*=1)
- *y*(0): Outcome without treatment (*D*=0)

分位数处理效应(QTE)



QTE under Exogeneity

- 如果处理变量*D*为外生，可直接对以下方程进行分位数回归

$$y_i = \alpha + D_i'\gamma + \mathbf{x}_i'\boldsymbol{\beta} + \varepsilon_i$$

- 其中，$\hat{\gamma}(\tau)$ 即为 τ 分位数的处理效应。

QTE under Endogeneity

- 如果处理变量*D*为内生，应进行工具变量法的分位数回归（IVQTE）

- 方法1： Abadie, A., J. Angrist, and G. Imbens. 2002. Instrumental variables estimates of the effect of subsidized training on the quantiles of trainee earnings. *Econometrica* 70: 91–117. 简称“AAI法”

- 允许异质性处理效应，但要求处理变量与工具变量均为虚拟变量

异质性工具变量法

- 传统的工具变量法假设同质的处理效应（homogeneous or constant treatment effects）

$$y_i = \alpha + \beta D_i + \varepsilon_i$$

- 但异质性处理效应（heterogeneous treatment effects）可能更接近于现实。比如，不同个体的教育回报率不同。

$$y_i = \alpha + \beta_i D_i + \varepsilon_i$$

- 假设处理变量 D 与工具变量 Z 皆为虚拟变量

Motivation: Experiments with Imperfect Compliance

Imperfect compliance in a randomized evaluation of a training program

	Enrolled in training	Not enrolled in training	Total
Assigned to training	4804	2683	7487
Assigned to control	54	3663	3717
Total	4858	6346	11204

- Consider the JTPA experiment, an experimental evaluation of a training program where many experimental subjects did not comply with the randomized assignment
- Units receiving training may differ from units that do not receive training
- Still, randomized assignment has an effect on the probability of receiving training
- Instrumental variables use the variation in receipt of training induced by the experiment to estimate of the effect of training

Randomized Experiments with Imperfect Compliance

Assignment

$$Z = \begin{cases} 1 & \text{if assigned to treatment group} \\ 0 & \text{if assigned to control group} \end{cases}$$

Potential Treatments

- D_1: treatment status if assigned to treatment group
- D_0: treatment status if assigned to control group

Observed Treatment

$$D = \begin{cases} D_1 & \text{if } Z = 1 \\ D_0 & \text{if } Z = 0 \end{cases}$$

or, in a more compact notation:

$$D = ZD_1 + (1 - Z)D_0.$$

Randomized Experiments with Imperfect Compliance

- Angrist, Imbens and Rubin (1996) define:
  - **Compliers**: $D_1 > D_0$ ($D_0 = 0$ and $D_1 = 1$)
  - **Always-takers**: $D_1 = D_0 = 1$
  - **Never-takers**: $D_1 = D_0 = 0$
  - **Defiers**: $D_1 < D_0$ ($D_0 = 1$ and $D_1 = 0$)
- Notice that for compliers, we still have a perfect experiment.
- However, only one of the potential treatment indicators, (D_0, D_1), is observed, so we cannot identify which group any particular individual belongs to.

基本问题

- 如果存在抗拒者(defiers)，则无法识别因果关系

- **原因**：在存在异质性处理效应的情况下，即使所有个体的处理效应均为正，但当Z从0变到1时，依从者(compliers)的正效应可能为抗拒者(defiers)的反向变动所抵消。

识别方法

- 假设不存在defiers，则 $D_{1i} \geq D_{0i}$, $\forall i$
- 这被称为“单调性假设”(monotonicity)，即 *Z* 对于 *D* 的作用方向对所有人都一样
- 理论上，如果 $D_{1i} \leq D_{0i}$, $\forall i$ 也可以（作用方向也一致），但很少见。故假设 $D_{1i} \geq D_{0i}$, $\forall i$

识别条件

The assumptions underlying the potential outcomes framework for IV are given below:

ASSUMPTION 2.1: *For almost all values of* X:

(i) INDEPENDENCE: (Y_1, Y_0, D_1, D_0) *is jointly independent of* Z *given* X.

(ii) NONTRIVIAL ASSIGNMENT: $P(Z=1|X) \in (0,1)$.

(iii) FIRST-STAGE: $E[D_1|X] \neq E[D_0|X]$.

(iv) MONOTONICITY: $P(D_1 \geq D_0|X) = 1$.

(i)：IV的外生性

(ii)与 (iii)：IV的相关性

(iv)：单调性

An Infeasible Approach

友万科技

- For compliers ($D_1 > D_0$), we still have a perfect experiment

$$(\alpha_\tau, \boldsymbol{\beta}_\tau) = \underset{\alpha, \boldsymbol{\beta}}{\arg\min}\, \mathrm{E}\left[\rho_\tau(y - \alpha D - \mathbf{x}'\boldsymbol{\beta}) \mid D_1 > D_0\right]$$

- 但由于不知道谁是依从者，无法求此条件期望 $\mathrm{E}[\cdot \mid D_1 > D_0]$，故此法不可行

A Feasible Approach

- 使用权重κ 进行校正

$$(\alpha_\tau, \boldsymbol{\beta}_\tau) = \arg\min_{\alpha, \boldsymbol{\beta}} \mathrm{E}\left[\kappa \rho_\tau (y - \alpha D - \mathbf{x}'\boldsymbol{\beta})\right]$$

- 其中，权重为

$$\kappa = 1 - \frac{D(1-Z)}{1-P(Z=1|\mathbf{x})} - \frac{(1-D)Z}{P(Z=1|\mathbf{x})} = \begin{cases} 1 & if\ \ D = Z \\ 1 - \dfrac{1}{1-P(Z=1|\mathbf{x})} & if\ \ D=1, Z=0 \\ 1 - \dfrac{(1-D)Z}{P(Z=1|\mathbf{x})} & if\ \ D=0, Z=1 \end{cases}$$

Noncovexity

- 虽然打钩函数 $\rho_\tau(y-\alpha D-\mathbf{x}'\boldsymbol{\beta})$ 为凸函数，但由于权重 κ 可能为负，故目标函数不再是凸函数，不便于进行最优化

- 定义 κ 的条件期望

$$\kappa_v = \mathrm{E}\left[\kappa \mid y, D, \mathbf{x}\right]$$

$$= 1 - \frac{D(1-\mathrm{E}(Z \mid y, D, \mathbf{x}))}{1-P(Z=1 \mid \mathbf{x})} - \frac{(1-D)\mathrm{E}(Z \mid y, D, \mathbf{x})}{P(Z=1 \mid \mathbf{x})} \geq 0$$

两步法估计

- 第一步，使用 logit, local logit 或 nonparametric power series estimator 估计 $P(Z_i=1|\mathbf{x}_i)$ 与 $\mathrm{E}(Z_i|y_i,D_i,\mathbf{x}_i)$， 得到 $\hat{\kappa}_{vi}$

- 第二步，求解最小化问题

$$(\alpha_\tau,\boldsymbol{\beta}_\tau)=\arg\min_{\alpha,\boldsymbol{\beta}}\frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(\hat{\kappa}_{vi}\ge 0)\cdot\hat{\kappa}_{vi}\cdot\rho_\tau(y_i-\alpha D_i-\mathbf{x}_i'\boldsymbol{\beta})$$

- 注：样本估计值 $\hat{\kappa}_{vi}$ 依然可能为负

IVQTE的Stata命令

- findit ivqte

```
ivqte depvar [indepvars] (treatment [= instrument]) [if] [in] [,
    quantiles(numlist) continuous(varlist) dummy(varlist) unordered(varlist)
    aai linear mata_opt kernel(kernel) bandwidth(#) lambda(#) trim(#)
    positive pbandwidth(#) plambda(#) pkernel(kernel) variance
    vbandwidth(#) vlambda(#) vkernel(kernel) level(#)
    generate_p(newvarname [, replace]) generate_w(newvarname [,
    replace]) phat(varname) what(varname)]
```

- If an instrument is provided and aai is activated, the estimator proposed by Abadie, Angrist, and Imbens (2002) is used.

案例 2、大学教育的投资回报

- Card, D. E. 1995. Using geographic variation in college proximity to estimate the return to schooling. In *Aspects of Labour Economics: Essays in Honour of John Vanderkamp*, ed. L. Christofides, E. K. Grant, and R. Swindinsky. Toronto, Canada: Univ. of Toronto Press.

- `use card.dta,clear`

- `des lwage college nearc4 exper black motheduc`

- Note: College proximity (nearc4) as IV for college education (college)

0.1分位数回归

variable name	storage type	display format	value label	variable label
lwage	float	%9.0g		log(wage)
college	float	%9.0g		
nearc4	byte	%9.0g		=1 if near 4 yr college, 1966
exper	byte	%9.0g		age - educ - 6
black	byte	%9.0g		=1 if black
motheduc	byte	%9.0g		mother's schooling

- qreg lwage college exper black motheduc reg662 reg663 reg664 reg665 reg666 reg667 reg668 reg669, quantile(0.1) nolog

```
.1 Quantile regression                              Number of obs =      2,657
  Raw sum of deviations 214.6026 (about 5.6801724)
  Min sum of deviations 194.8344                    Pseudo R2     =     0.0921
```

lwage	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
college	.2064537	.0342966	6.02	0.000	.1392028	.2737046
exper	.0217235	.0041486	5.24	0.000	.0135887	.0298583
black	-.2316504	.0395452	-5.86	0.000	-.3091931	-.1541077
motheduc	.0131027	.0050343	2.60	0.009	.0032311	.0229743
reg662	.0867458	.0752728	1.15	0.249	-.0608537	.2343453
reg663	.2204661	.0741461	2.97	0.003	.0750758	.3658563
reg664	.0132414	.0867677	0.15	0.879	-.1568981	.1833809
reg665	.0266135	.0757109	0.35	0.725	-.1218452	.1750721
reg666	-.0337565	.0837077	-0.40	0.687	-.1978956	.1303827
reg667	-.0069512	.0804102	-0.09	0.931	-.1646245	.1507221
reg668	-.0388916	.1059618	-0.37	0.714	-.2466681	.1688848
reg669	.0961543	.0814656	1.18	0.238	-.0635885	.255897
_cons	5.307295	.1017405	52.17	0.000	5.107796	5.506794

异方差稳健的标准误

- ivqte lwage exper black motheduc reg662 reg663 reg664 reg665 reg666 reg667 reg668 reg669 (college), quantiles(0.1) variance

- 也可使用自助标准误，但因样本容量大，较费时：

- bsqreg lwage college exper black motheduc reg662 reg663 reg664 reg665 reg666 reg667 reg668 reg669, quantile(0.1) reps(500)

```
Quantile regression
Estimator suggested in Koenker and Bassett (1978)

Quantile:                    .1
Dependent variable:          lwage
Regressor(s):                college exper black motheduc reg662 reg663 reg664 reg665 reg666 r
Number of observations:      2657
```

lwage	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
college	.2064537	.0353578	5.84	0.000	.1371537	.2757536
exper	.0217235	.0038234	5.68	0.000	.0142299	.0292172
black	-.2316504	.0371132	-6.24	0.000	-.304391	-.1589098
motheduc	.0131027	.0050099	2.62	0.009	.0032834	.022922
reg662	.0867458	.0801968	1.08	0.279	-.0704369	.2439286
reg663	.2204661	.0762157	2.89	0.004	.071086	.3698461
reg664	.0132414	.0853672	0.16	0.877	-.1540752	.180558
reg665	.0266135	.0758597	0.35	0.726	-.1220688	.1752958
reg666	-.0337565	.083806	-0.40	0.687	-.1980132	.1305003
reg667	-.0069512	.0796822	-0.09	0.930	-.1631255	.1492231
reg668	-.0388916	.1108855	-0.35	0.726	-.2562233	.17844
reg669	.0961543	.0924436	1.04	0.298	-.0850319	.2773404
_cons	5.307295	.0992185	53.49	0.000	5.11283	5.501759

IVQTE (AAI, 2002)

- `ivqte lwage (college=nearc4), quantiles(0.1) variance dummy(black) continuous(exper motheduc) unordered(region) aai`

- 使用logit估计 $P(Z_i = 1 | \mathbf{x}_i)$ 与 $\mathrm{E}(Z_i | y_i, D_i, \mathbf{x}_i)$

```
IV quantile regression
Estimator suggested in Abadie, Angrist and Imbens (2002)

Quantile(s):                 .1
Dependent variable:          lwage
Treatment variable:          college
Instrumental variable:       nearc4
Control variable(s):         exper motheduc black region
Number of observations:      2657
Proportion of compliers:     .091

Propensity score estimated by local logit regression with h = infinity and lambda = 1
Positive weights estimated by local linear regression with h = infinity and lambda = 1
Variance estimated using local linear regression with h = infinity and lambda = 1
```

lwage	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
college	.7779974	.2635231	2.95	0.003	.2615015	1.294493
exper	.0166582	.0756843	0.22	0.826	-.1316802	.1649967
motheduc	.0095984	.0655372	0.15	0.884	-.1188521	.1380489
black	-.1502784	.6064568	-0.25	0.804	-1.338912	1.038355
region2	-.0308088	1.183632	-0.03	0.979	-2.350684	2.289067
region3	.10724	1.204884	0.09	0.929	-2.254288	2.468768
region4	.0493495	1.399898	0.04	0.972	-2.694399	2.793098
region5	-.0316856	1.154963	-0.03	0.978	-2.295372	2.232001
region6	-.0077108	1.454662	-0.01	0.996	-2.858796	2.843375
region7	-.1112334	1.283061	-0.09	0.931	-2.625988	2.403521
region8	.1253792	2.581608	0.05	0.961	-4.93448	5.185239
region9	.028913	1.345531	0.02	0.983	-2.60828	2.666106
_cons	5.136558	1.370468	3.75	0.000	2.450491	7.822626

Nonparametric First Step

- 使用local logit估计 $P(Z_i = 1 | \mathbf{x}_i)$ 与 $\mathrm{E}(Z_i | y_i, D_i, \mathbf{x}_i)$

- 使用留一法（leave-one-out cross-validation）选择最优窗口 h 与平滑参数 $0 \le \lambda \le 1$ (平衡连续变量与离散变量)

- 因计算量大，仅随机选取500观测值

第一步

- `set seed 123456`
- `gen ranorder = runiform()`
- `sort ranorder`
- `gen sample=(_n<=500)`

- `locreg nearc4 if sample, dummy(black) continuous(exper) unordered(region) bandwidth(0.5 0.8) lambda(0.8 1) generate(ps) logit`

- 注：仅考虑 $h = 0.5$或0.8, $\lambda = 0.8$或1，将估计结果存为ps。locreg为命令ivqte自带的子命令

最优带宽与平滑参数

- 估计 $P(Z_i = 1 | \mathbf{x}_i)$

```
Leave-one-out cross-validation
```

Bandwidth	Lambda	Mean Squared Error
.5	.8	3.3565862
.5	1	3.3504882
.8	.8	3.0871657
.8	1	3.0924464

```
Among the grid of values tested, the optimal bandwidth is .8 and the optimal lambda is .8.
```

- 估计 κ ：gen waai=1-college*(1-nearc4)/(1-ps)-(1-college)*nearc4/ps if sample

通过回归计算*kappa*的条件期望

- 对处理组与控制组分别估计 $\hat{\kappa}_{vi}$，建议选择最小的带宽与平滑参数
- `locreg waai if college==1 & sample==1, dummy(black) continuous(exper motheduc lwage) unordered(region) bandwidth(0.5 0.8) lambda(0.8 1)`

```
Leave-one-out cross-validation
```

Bandwidth	Lambda	Mean Squared Error
.5	.8	3.5375789
.5	1	3.5190053
.8	.8	3.4005234
.8	1	3.4031062

```
Among the grid of values tested, the optimal bandwidth is .8 and the optimal lambda is .8.
```

通过回归计算*kappa*的条件期望(2)

- `locreg waai if college==0 & sample==1, dummy(black) continuous(exper motheduc lwage) unordered(region) bandwidth(0.5 0.8) lambda(0.8 1)`

Leave-one-out cross-validation

Bandwidth	Lambda	Mean Squared Error
.5	.8	.84530862
.5	1	.84479758
.8	.8	.87549462
.8	1	.87395397

Among the grid of values tested, the optimal bandwidth is .5 and the optimal lambda is 1.

第二步：IVQTE估计

- `ivqte lwage (college=nearc4) if sample, aai quantiles(0.1) variance continuous(exper motheduc) unordered(region) dummy(black) bandwidth(0.8) lambda(0.8) pbandwidth(0.5) plambda(0.8)`

```
42 observations have been trimmed. 400 observations are left after trimming.

IV quantile regression
Estimator suggested in Abadie, Angrist and Imbens (2002)

Quantile(s):                 .1
Dependent variable:          lwage
Treatment variable:          college
Instrumental variable:       nearc4
Control variable(s):         exper motheduc black region
Number of observations:      442
Proportion of compliers:     .228

Propensity score estimated by local logit regression with h = 0.8 and lambda = .8
Positive weights estimated by local linear regression with h = 0.5 and lambda = 0.8
Variance estimated using local linear regression with h = .8 and lambda = .8
```

lwage	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
college	.7036065	.2162667	3.25	0.001	.2797317	1.127481
exper	.0531363	.0202505	2.62	0.009	.013446	.0928267
motheduc	-.0392274	.0112194	-3.50	0.000	-.061217	-.0172377
black	-.2131487	.1151599	-1.85	0.064	-.4388579	.0125605
region2	.0793567	.1235338	0.64	0.521	-.1627651	.3214784
region3	.1849337	.3096847	0.60	0.550	-.4220371	.7919045
region4	-.0200778	.1353394	-0.15	0.882	-.2853382	.2451826
region5	.082572	.1599221	0.52	0.606	-.2308697	.3960136
region6	.2847685	.452841	0.63	0.529	-.6027835	1.172321
region7	.0596625	.1468184	0.41	0.684	-.2280963	.3474213
region8	.8587742	.062021	13.85	0.000	.7372153	.9803331
region9	-.0426573	.1331519	-0.32	0.749	-.3036302	.2183157
_cons	5.162572	.3884685	13.29	0.000	4.401188	5.923956

中位数 IVQTE

- ivqte lwage (college=nearc4) if sample, aai quantiles(0.5) variance continuous(exper motheduc) unordered(region) dummy(black) bandwidth(0.8) lambda(0.8) pbandwidth(0.5) plambda(0.8)

```
85 observations have been trimmed. 366 observations are left after trimming.

IV quantile regression
Estimator suggested in Abadie, Angrist and Imbens (2002)

Quantile(s):                 .5
Dependent variable:          lwage
Treatment variable:          college
Instrumental variable:       nearc4
Control variable(s):         exper motheduc black region
Number of observations:      451
Proportion of compliers:     .282

Propensity score estimated by local logit regression with h = 0.8 and lambda = .8
Positive weights estimated by local linear regression with h = 0.5 and lambda = 0.8
Variance estimated using local linear regression with h = .8 and lambda = .8
```

lwage	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
college	.2523874	.084603	2.98	0.003	.0865685	.4182063
exper	.0120726	.0139396	0.87	0.386	-.0152486	.0393937
motheduc	-.0042144	.0189828	-0.22	0.824	-.0414201	.0329912
black	-.3016094	.1950533	-1.55	0.122	-.6839067	.080688
region2	.0428996	.2150836	0.20	0.842	-.3786566	.4644558
region3	-.0425843	.2102968	-0.20	0.840	-.4547583	.3695898
region4	-.4544888	.2181627	-2.08	0.037	-.8820798	-.0268977
region5	-.2643078	.4299554	-0.61	0.539	-1.107005	.5783893
region6	-.518493	.2838161	-1.83	0.068	-1.074762	.0377763
region7	-.1227452	.2504194	-0.49	0.624	-.6135581	.3680677
region8	-.548141	.2510207	-2.18	0.029	-1.040133	-.0561495
region9	-.0368156	.1975456	-0.19	0.852	-.4239978	.3503666
_cons	6.253284	.33502	18.67	0.000	5.596657	6.909911

命令ivqte的更多细节

- Frölich, M., and B. Melly. 2010. Estimation of quantile treatment effects with Stata. *Stata Journal* 10: 423-457.
- 该命令还可进行“无条件分位数回归”(unconditional quantile regression)，即考察 x 变动对 y 的无条件(边际)分布的影响

AAI法的缺点

- 要求处理变量*D*与工具变量Z均为虚拟变量
- 仅能识别依从者（compliers）的平均处理效应

QTE under Endogeneity (2)

- 方法 2：Chernozhukov and Hansen (2005, 2006, 2008)，简称“CH法”
- 优点：不要求处理变量或工具变量为虚拟变量；可识别整个样本的平均处理效应
- 缺点：须额外假设“排名不变性”(rank invariance)或“排名相似性”(rank similarity)

参考文献

- [1] Chernozhukov, V. and Hansen, C. 2005. An IV model of quantile treatment effects. *Econometrica* 73 (1) p.379-398. 模型与识别条件

- [2] Chernozhukov, V. and Hansen, C. 2006. Instrumental quantile regression inference for structural and treatment effect models. *Journal of Econometrics* 132 (2) p.491-525. 恰好识别情形下的估计

- [3] Chernozhukov, V. and Hansen, C. 2008. Instrumental variable quantile regression: A robust inference approach. *Journal of Econometrics* 142 (1) p.379-398. 过度识别情形下的估计

- [4] Chernozhukov, V., C. Hansen and Jansson M. 2007. Inference approaches for instrumental variable quantile regression. *Economics Letters* 95, 272-277. 应用与比较，弱工具变量问题

排名不变性或相似性

- “排名不变性”(rank invariance)：在D=0的世界里，给定协变量 $\boldsymbol{x}$，如果个体的结果变量 y 排在第 τ 分位数，则在D=1世界里也排在第 τ 分位数

- “排名相似性”(rank similarity)：不要求排在完全相同的分位数，但二者排位的分布相同

识别条件

- 记 $q(D,\mathbf{x},\tau)$ 为quantile treatment response function。如果*D*为外生：

$$P\left[y \le q(D,\mathbf{x},\tau) \mid \mathbf{x}\right] = \tau$$

- 如果*D*为内生，而*Z*为工具变量：

$$P\left[y \le q(D,\mathbf{x},\tau) \mid \mathbf{x}, Z\right] = \tau$$

- 利用此条件进行两阶段回归。也可将此条件视为矩条件

CH法的Stata命令

- Stata命令 `ivqreg` 下载地址：http://faculty.chicagobooth.edu/christian.hansen/research/ivqrstata.zip （用命令`sysdir`找到并存入 plus文件夹）

- 另一实现方法 (矩估计)：`ssc install ivqreg2`

例：Card (1995)

- `ivqreg lwage exper black motheduc reg662-reg669 (college = nearc4),q(0.1)`

- `ivqreg lwage exper black motheduc reg662-reg669 (college = nearc4),q(0.5)`

.1th Instrumental Variable Quantile Regression Number of obs = 2657

lwage	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
college	.7135388	.0342818	20.81	0.000	.6463477	.7807299
exper	.0474708	.0039292	12.08	0.000	.0397697	.055172
black	-.1224297	.0421702	-2.90	0.004	-.2050817	-.0397777
motheduc	-.0050838	.0052409	-0.97	0.332	-.0153557	.0051882
reg662-reg669	.2035439	.0802568	2.54	0.011	.0462435	.3608443
_cons	.229825	.0791339	2.90	0.004	.0747255	.3849245
_cons	.0950028	.0928706	1.02	0.306	-.0870202	.2770258
_cons	.0898886	.0811548	1.11	0.268	-.0691719	.2489491
_cons	-.0568914	.0910342	-0.62	0.532	-.2353152	.1215324
_cons	.0379809	.0862706	0.44	0.660	-.1311062	.2070681
_cons	-.0054157	.1129133	-0.05	0.962	-.2267217	.2158903
_cons	.1096704	.0867055	1.26	0.206	-.0602694	.2796101
_cons	4.855065	.1096594	44.27	0.000	4.640137	5.069994

.5th Instrumental Variable Quantile Regression Number of obs = 2657

lwage	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
college	1.23566	.0321919	38.38	0.000	1.172565	1.298755
exper	.0934182	.0036897	25.32	0.000	.0861865	.1006498
black	-.1447192	.0395993	-3.65	0.000	-.2223325	-.067106
motheduc	-.0172861	.0049214	-3.51	0.000	-.0269318	-.0076404
reg662-reg669	-.0576607	.0753641	-0.77	0.444	-.2053715	.0900501
_cons	-.0271864	.0743096	-0.37	0.714	-.1728305	.1184577
_cons	-.214984	.0872089	-2.47	0.014	-.3859103	-.0440577
_cons	-.2737779	.0762074	-3.59	0.000	-.4231415	-.1244142
_cons	-.2609591	.0854845	-3.05	0.002	-.4285056	-.0934126
_cons	-.1957343	.0810112	-2.42	0.016	-.3545134	-.0369552
_cons	-.3929353	.1060297	-3.71	0.000	-.6007497	-.1851208
_cons	-.1901273	.0814197	-2.34	0.020	-.3497069	-.0305476
_cons	5.163871	.1029742	50.15	0.000	4.962045	5.365696

Implementation by `ivqreg2`

- 使用 Machado and Silva (2019)的矩估计 (MM-QR)

- `ivqreg2 lwage college exper black motheduc reg662-reg669,inst(nearc4 exper black motheduc reg662-reg669) quantile(0.1)`

- `ivqreg2 lwage college exper black motheduc reg662-reg669,inst(nearc4 exper black motheduc reg662-reg669) quantile(0.5)`

MM-QR regression results

Number of obs = 2657
.1 Structural quantile function

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
college	.620435	1.487187	0.42	0.677	-2.294398	3.535268
exper	.0588703	.0201836	2.92	0.004	.0193111	.0984294
black	-.161785	.1128107	-1.43	0.152	-.3828899	.0593199
motheduc	-.0054225	.0151216	-0.36	0.720	-.0350603	.0242153
reg662	.0485315	.1531046	0.32	0.751	-.2515481	.3486111
reg663	.046059	.0863041	0.53	0.594	-.123094	.215212
reg664	-.1232738	.1682646	-0.73	0.464	-.4530663	.2065187
reg665	-.1172158	.2063293	-0.57	0.570	-.5216138	.2871821
reg666	-.1881112	.2277591	-0.83	0.409	-.6345108	.2582884
reg667	-.0848297	.1875891	-0.45	0.651	-.4524977	.2828382
reg668	-.2302179	.3586201	-0.64	0.521	-.9331004	.4726647
reg669	-.0625742	.2818079	-0.22	0.824	-.6149075	.4897591
_cons	4.994454	.3864103	12.93	0.000	4.237103	5.751804

Number of obs = 2657
.5 Structural quantile function

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
college	1.179498	.9975035	1.18	0.237	-.7755726	3.134569
exper	.0754614	.0241462	3.13	0.002	.0281356	.1227871
black	-.1514182	.0814075	-1.86	0.063	-.310974	.0081376
motheduc	-.0143331	.0194185	-0.74	0.460	-.0523927	.0237265
reg662	.0396527	.1319009	0.30	0.764	-.2188683	.2981737
reg663	.0651144	.0740624	0.88	0.379	-.0800453	.2102741
reg664	-.0934962	.1295218	-0.72	0.470	-.3473542	.1603619
reg665	-.1530219	.1730391	-0.88	0.377	-.4921723	.1861286
reg666	-.1784179	.1871285	-0.95	0.340	-.545183	.1883473
reg667	-.123381	.1629336	-0.76	0.449	-.442725	.195963
reg668	-.281804	.2981869	-0.95	0.345	-.8662395	.3026315
reg669	-.0641362	.2296811	-0.28	0.780	-.5143028	.3860305
_cons	5.209834	.2700064	19.30	0.000	4.680631	5.739036

面板分位数回归

- 方法1：Powell (2016) (still unpublished)

```
ssc install moremata
ssc install qregpd
注：命令 qregpd 须调用 moremata
```

- 方法2：MM-QR by Machado and Silva (2019, JOE)

```
ssc install xtqreg
```

MM-QR的识别条件：

- 假设协变量 x 仅影响 y 的一阶矩(location)与二阶矩(scale)：

$$y = \alpha + \mathbf{x}'\boldsymbol{\beta} + \sigma(\delta + \mathbf{z}'\boldsymbol{\gamma})u$$

- 其中，$\mathbf{z}$ 为 $\mathbf{x}$ 的函数（比如 $\mathbf{z} = \mathbf{x}$），而 $\sigma(\cdot)$ 为已知函数（比如 $\sigma(\cdot) \equiv 1$）。

- 由此 Location-scale Model 得到矩条件，进行矩估计（Method of Moments）

面板固定效应 MM-QR

- 考虑如下 location-scale panel model

$$y_{it} = \alpha_i + \mathbf{x}_{it}'\boldsymbol{\beta} + (\delta_i + \mathbf{z}_{it}'\boldsymbol{\gamma})u_{it}$$

- 其条件分位数函数为

$$Q_y(\tau \mid \mathbf{x}_{it}) = \alpha_i + \delta_i q(\tau) + \mathbf{x}_{it}'\boldsymbol{\beta} + \mathbf{z}_{it}'\boldsymbol{\gamma} q(\tau)$$

- 进行相应的矩估计

例：交通死亡率

- use traffic.dta.clear
- xtset state year
- 作为参照系，先进行常规的固定效应估计
- xtreg fatal beertax spircons unrate perinck,fe r

```
Fixed-effects (within) regression               Number of obs      =       336
Group variable: state                           Number of groups   =        48

R-sq:                                           Obs per group:
     within  = 0.3526                                         min =         7
     between = 0.1146                                         avg =       7.0
     overall = 0.0863                                         max =         7

                                                F(4,47)            =     21.27
corr(u_i, Xb)  = -0.8804                        Prob > F           =    0.0000

                                  (Std. Err. adjusted for 48 clusters in state)
------------------------------------------------------------------------------
             |               Robust
       fatal |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     beertax |  -.4840728   .2218754    -2.18   0.034    -.9304285    -.037717
    spircons |   .8169652   .1272627     6.42   0.000     .5609456    1.072985
      unrate |  -.0290499   .0094581    -3.07   0.004    -.0480772   -.0100227
     perinck |   .1047103   .0341455     3.07   0.004     .0360184    .1734022
       _cons |   -.383783   .7091738    -0.54   0.591    -1.810457    1.042891
-------------+----------------------------------------------------------------
     sigma_u |  1.1181913
     sigma_e |  .15678965
         rho |  .98071823   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

面板固定效应中位数回归

- xtqreg fatal beertax spircons unrate perinck,i(state) quantile(0.5)

```
                              MM-QR regression results
Number of obs = 336
.5 Quantile regression
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
beertax	-.4844695	.1525989	-3.17	0.001	-.7835578	-.1853812
spircons	.8169576	.0940864	8.68	0.000	.6325516	1.001364
unrate	-.0290526	.0080348	-3.62	0.000	-.0448005	-.0133047
perinck	.1046758	.0210479	4.97	0.000	.0634226	.1459289

面板固定效应分位数回归

- 同时进行多个分位数回归

- `xtqreg fatal beertax spircons unrate perinck,i(state) quantile(.1(0.1)0.9)`

MM-QR regression results

Number of obs = 336

.1 Quantile regression

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
beertax	-.3846008	.2777339	-1.38	0.166	-.9289493	.1597477
spircons	.8188633	.1713929	4.78	0.000	.4829394	1.154787
unrate	-.0283849	.0146363	-1.94	0.052	-.0570716	.0003019
perinck	.1133645	.0383275	2.96	0.003	.0382441	.188485

.2 Quantile regression

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
beertax	-.4115626	.2275472	-1.81	0.070	-.8575469	.0344217
spircons	.8183488	.140398	5.83	0.000	.5431738	1.093524
unrate	-.0285651	.0119895	-2.38	0.017	-.0520642	-.0050661
perinck	.1110188	.0313986	3.54	0.000	.0494786	.172559

.3 Quantile regression

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
beertax	-.4397066	.1839801	-2.39	0.017	-.8003009	-.0791122
spircons	.8178117	.1134772	7.21	0.000	.5954006	1.040223
unrate	-.0287533	.0096907	-2.97	0.003	-.0477466	-.00976
perinck	.1085702	.0253818	4.28	0.000	.0588228	.1583177

.4 Quantile regression

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
beertax	-.4612572	.1612469	-2.86	0.004	-.7772954	-.1452191
spircons	.8174005	.0994488	8.22	0.000	.6224845	1.012317
unrate	-.0288974	.0084927	-3.40	0.001	-.0455427	-.012252
perinck	.1066953	.0222447	4.80	0.000	.0630965	.1502941

.5 Quantile regression

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
beertax	-.4844695	.1525989	-3.17	0.001	-.7835578	-.1853812
spircons	.8169576	.0940864	8.68	0.000	.6325516	1.001364
unrate	-.0290526	.0080348	-3.62	0.000	-.0448005	-.0133047
perinck	.1046758	.0210479	4.97	0.000	.0634226	.1459289

.6 Quantile regression

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
beertax	-.5067528	.1620036	-3.13	0.002	-.8242741	-.1892315
spircons	.8165324	.0999123	8.17	0.000	.6207078	1.012357
unrate	-.0292016	.0085323	-3.42	0.001	-.0459245	-.0124786
perinck	.1027371	.0223487	4.60	0.000	.0589345	.1465397

.7 Quantile regression

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
beertax	-.5243853	.1801553	-2.91	0.004	-.8774833	-.1712874
spircons	.8161959	.1111446	7.34	0.000	.5983566	1.034035
unrate	-.0293195	.0094914	-3.09	0.002	-.0479223	-.0107166
perinck	.101203	.0248576	4.07	0.000	.052483	.149923

.8 Quantile regression

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
beertax	-.5520676	.2216929	-2.49	0.013	-.9865776	-.1175575
spircons	.8156677	.1367705	5.96	0.000	.5476024	1.083733
unrate	-.0295045	.0116798	-2.53	0.012	-.0523965	-.0066126
perinck	.0987946	.0305888	3.23	0.001	.0388416	.1587476

.9 Quantile regression

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
beertax	-.5859877	.284722	-2.06	0.040	-1.144033	-.0279428
spircons	.8150204	.1757094	4.64	0.000	.4706364	1.159404
unrate	-.0297313	.015005	-1.98	0.048	-.0591405	-.0003222
perinck	.0958435	.0392924	2.44	0.015	.0188319	.1728551

MM-QR的缺点

- 须假设 location-scale model
- 目前只能估计单向固定效应，不能估计双向固定效应（无法放入时间固定效应）